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But

$$\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - \dots$$

Hence,

$$\frac{d}{dx} \log [(\sin x)/x] = -\frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} - \dots$$

from which by integration, we get

$$\log [(\sin x)/x] = -\frac{x^2}{6} - \frac{x^4}{180} - \frac{x^6}{2835} - \dots$$

A solution similar to I was received from LEVY M. COFFIN.

GEOMETRY.

423. Proposed by C. N. SCHMALL, New York City.

The sum of the squares of the distances of a point from n fixed points is constant. Show that the locus of the point is a circle.

SOLUTION BY W. C. EELLS, Tacoma, Wash.

Let $(x_1, y_1), (x_2, y_2) \dots (x_n, y_n)$ be the coördinates of the n fixed points, and (x, y) of the moving point. Since $d_i = \sqrt{(x - x_i)^2 + (y - y_i)^2}$, the given condition may be expressed,

$$(x - x_1)^2 + (y - y_1)^2 + (x - x_2)^2 + (y - y_2)^2 + \dots + (x - x_n)^2 + (y - y_n)^2 = k,$$

which reduces to

$$\begin{aligned} nx^2 + ny^2 - 2(x_1 + x_2 + \dots + x_n)x - 2(y_1 + y_2 + \dots + y_n)y \\ + (x_1^2 + x_2^2 + \dots + x_n^2) + (y_1^2 + y_2^2 + \dots + y_n^2) - k = 0. \end{aligned}$$

This may be expressed more briefly,

$$x^2 + y^2 - \left(\frac{2}{n} \sum_{i=1}^n x_i \right) x - \left(\frac{2}{n} \sum_{i=1}^n y_i \right) y + \frac{1}{n} \left(\sum_{i=1}^n x_i^2 + \sum_{i=1}^n y_i^2 - k \right) = 0.$$

This is a circle, with center at $\left(\frac{1}{n} \sum_{i=1}^n x_i, \frac{1}{n} \sum_{i=1}^n y_i \right)$, the square of whose radius is

$$\left(\frac{1}{n} \sum_{i=1}^n x_i \right)^2 + \left(\frac{1}{n} \sum_{i=1}^n y_i \right)^2 - \frac{1}{n} \left(\sum_{i=1}^n x_i^2 + \sum_{i=1}^n y_i^2 - k \right).$$

Similar solutions were received from H. C. FEEMSTER, S. W. REAVES, F. M. MORGAN, ELMER SCHUYLER, and C. N. SCHMALL.

A solution of 418 was received from D. F. KELLEY; one of 420 from C. N. SCHMALL, and one of 422 from H. E. TREFETHEN, after the list of solutions was made up and sent to the printer.

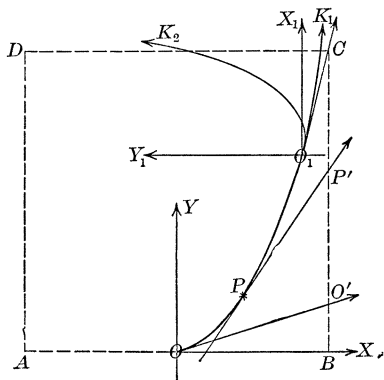
CALCULUS.

327. Proposed by RICHARD P. LOCHNER, Philadelphia, Penn.

A hound is at the middle point of the side of a square field, and a fox is at an adjacent corner. How far will the hound run to catch the fox if the fox runs on the perimeter of the field and the hound runs directly towards the fox at all times, the hound running n times as fast as the fox? Where will the race end?

SOLUTION BY H. EGERER, Wilkensburg, Pa.

The hound is running on a curve, which has the property that the length of the curve between two points is proportional to the distance cut off by the tangents at these two points on a straight line.



To find this curve $y = f(x)$, which we call K , we take two perpendicular axes through a point O of the curve, so that the y -axis is parallel to the given straight line BC . Furthermore, O' , the starting point of the fox, is given by $OB = a$, $O'B = b$. Then we have

$$\text{arc } OP = n \cdot O'P'.$$

Now

$$\text{arc } OP = \int_0^x \sqrt{1 + y'^2} dx, \quad O'P' = y + y'(a - x) - b,$$

so that we have

$$y - b + y'(a - x) = \frac{1}{n} \int_0^x \sqrt{1 + y'^2} dx.$$

By differentiation, we get

$$y''(a - x) = \frac{1}{n} \sqrt{1 + y'^2}.$$

If we assume $y' = p$, hence $y'' = \frac{dp}{dx}$, we have $\frac{dx}{n(a - x)} = \frac{dp}{\sqrt{1 + p^2}}$,

and by integration

$$c(a - x)^{1/n} = p + \sqrt{1 + p^2},$$

or

$$p = \frac{dy}{dx} = \frac{c}{2} (a - x)^{1/n} - \frac{1}{2c} (a - x)^{-1/n},$$

and again by integration

$$y = y_a + \frac{n}{2c(n - 1)} (a - x)^{(n-1)/n} - \frac{cn}{2(n + 1)} (a - x)^{(n+1)/n}.$$

Since when $x = y = 0$, $p = \frac{b}{a}$, therefore $c = \frac{b + \sqrt{a^2 + b^2}}{a^{(n+1)/n}}$.

Hence, also

$$y_a = \frac{cn}{2(n+1)} a^{(n+1)/n} - \frac{n}{2c(n-1)} a^{(n-1)/n}.$$

Therefore, finally, we have

$$y = \frac{cn}{2(n+1)} (a^{(n+1)/n} - (a-x)^{(n+1)/n}) - \frac{n}{2c(n-1)} (a^{(n-1)/n} - (a-x)^{(n-1)/n}).$$

We see that the curve K_1 cuts BC at the point (a, y_a) and, since for $n = 1$ $y_a = \infty$, it follows that n must be greater than 1 if the hound can catch the fox.

If we assume the side of the square AB equal to 2, then we have $a = 1$, $b = 0$ and therefore $c = \pm 1$, where we have to take $c = -1$, because y' is always positive and we get

$$y = \frac{n}{2(n-1)} (1 - (1-x)^{(n-1)/n}) - \frac{n}{2(n+1)} (1 - (1-x)^{(n+1)/n}),$$

and

$$y_a = \frac{n}{2(n-1)} - \frac{n}{2(n+1)}.$$

As long as $y_a < 2$, the hound catches the fox on the side BC of the square, and we find the smallest n from $2 = \frac{n}{2(n-1)} - \frac{n}{2(n+1)}$, that is, $n = 1.3680$. If $n < 1.3680$, then we find the point $O_1 \equiv (x_1, y_1)$ whose tangent passes through C and lay a new system of axes X_1Y_1 for the curve K_2 . Here is $a = 2 - y_1$, $b = 1 - x_1$, so that we find the equation of the curve K_2 and can find y_a in which alone we are interested. If the hound does not catch the fox on the side CD we have to repeat the process.

We find O_1 by use of the equation $2 = y + y'(a-x)$.

By substituting the above values for y and y' , we find

$$\frac{2n}{n^2-1} - 4 = \frac{(1-x)^{(n+1)/n}}{n+1} + \frac{(1-x)^{(n-1)/n}}{n-1}.$$

If we assume $(1-x)^{1/n} = U$, we have the form $a = bU^m + cU^{m'}$ from which we find U , and hence x and O_1 .

Also solved by BARNUM LIBBY.

334. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.

Solve

$$\frac{\partial^2 T}{\partial u \partial v} + \frac{2v}{u^2 + v^2 + 1} \frac{\partial T}{\partial u} + \frac{2u}{u^2 + v^2 + 1} \frac{\partial T}{\partial v} = 0.$$